

GUIDE 01

Quadratic functions

Read a quadratic in multiple forms. Each form reveals a different feature of the same graph: vertex, zeros, y-intercept, or direction of opening.

Quadratic essentials

Connect form to graph features.

CONCEPT	FORMULA OR RULE	USE IT WHEN
Standard form	$f(x) = ax^2 + bx + c$	Use for the y-intercept, quadratic formula, and determining whether the parabola opens up or down.
Vertex form	$f(x) = a(x - h)^2 + k$	The vertex is (h, k). Use it to see transformations and maximum or minimum value.
Factored form	$f(x) = a(x - r_1)(x - r_2)$	The zeros are r_1 and r_2 . Use it to see x-intercepts and solve $f(x) = 0$.
Axis of symmetry	$x = -b / (2a)$	The vertex lies on this line. Substitute this x-value into $f(x)$ to find the vertex y-value.
Quadratic formula	$x = (-b \pm \sqrt{b^2 - 4ac}) / (2a)$	Use when a quadratic does not factor quickly. Substitute a, b, and c with their signs.

Worked check

For $f(x) = (x - 2)^2 - 3$, the vertex is (2, -3), the axis is $x = 2$, and the graph shifts right 2 and down 3.

GUIDE 02

Polynomials and complex numbers

Use structure before expanding. Factoring, division, and the remainder theorem all become easier when powers are written in descending order.

Polynomial methods

Check the divisor sign before using synthetic division.

CONCEPT	FORMULA OR RULE	USE IT WHEN
Remainder theorem	$P(c)$ = remainder from division by $(x - c)$	Evaluate $P(c)$ to find the remainder without completing long division.
Factor theorem	$P(c) = 0$ means $(x - c)$ is a factor	Use possible rational zeros, then divide to reduce a high-degree polynomial.
Synthetic division	Use c for divisor $(x - c)$	Fast method for dividing a polynomial by a linear binomial.
End behavior	Leading term controls far-left and far-right graph	Look at degree and leading coefficient to predict which direction each end moves.
Imaginary unit	$i^2 = -1$	Use when a square root of a negative number appears in a quadratic solution.

Complex number pattern

Powers of i repeat every four: i , -1 , $-i$, 1 . Reduce the exponent modulo 4 before simplifying.

GUIDE 03

Rational expressions and radicals

Restrictions matter. State where the original expression is undefined before you simplify, cancel, graph, or solve a rational equation.

Restrictions and roots

Restrictions come from the original expression.

CONCEPT	FORMULA OR RULE	USE IT WHEN
Domain restriction	Denominator cannot equal 0	Record excluded values from the original denominator before cancellation.
Vertical asymptote	Denominator = 0 after simplification	A non-cancelled denominator factor creates a vertical asymptote.
Horizontal asymptote	Compare numerator and denominator degrees	Lower degree gives $y = 0$; equal degree gives ratio of leading coefficients.
Rational exponent	$a^{(m/n)} = (\text{nth root of } a)^m$	Use the denominator as the root and the numerator as the power.
Radical equation	Isolate radical, square, then check	Squaring can create an extraneous answer, so test every result in the original equation.

Fast check

After multiplying a rational equation by an LCD, test each candidate. A value that makes any original denominator 0 cannot be a solution.

GUIDE 04

Exponential and logarithmic functions

Exponential and logarithmic functions are inverses. Use their graph behavior and inverse relationship to choose a method before manipulating an equation.

Growth, decay, and logs

Every logarithm needs a positive argument.

CONCEPT	FORMULA OR RULE	USE IT WHEN
Exponential model	$y = a(b^x)$	a is the initial amount. For growth $b > 1$; for decay $0 < b < 1$.
Continuous growth	$A = Pe^{(rt)}$	Use e when growth or decay is continuous rather than based on regular periods.
Log definition	$\log_b(x) = y$ means $b^y = x$	Rewrite between forms to solve or evaluate a logarithm.
Product rule	$\log_b(MN) = \log_b(M) + \log_b(N)$	Use to expand a product or condense a sum of logarithms.
Change of base	$\log_b(x) = \log(x) / \log(b)$	Use calculator logs to evaluate a logarithm with another base.

Graph cue

Exponential graphs have horizontal asymptotes. Logarithmic graphs have vertical asymptotes and only accept positive inputs.

GUIDE 05

Sequences, series, and modeling

A sequence describes a pattern term by term. A series adds those terms. Decide whether the change is constant, multiplied, or modeled by a function.

Patterns and models

Name the variable and units in every model.

CONCEPT	FORMULA OR RULE	USE IT WHEN
Arithmetic sequence	$a_n = a_1 + (n - 1)d$	Use when the same difference d is added each step.
Arithmetic series	$S_n = n / 2 [2a_1 + (n - 1)d]$	Use to add the first n terms of an arithmetic sequence.
Geometric sequence	$a_n = a_1(r)^{(n - 1)}$	Use when each term is multiplied by the same ratio r .
Geometric series	$S_n = a_1(1 - r^n) / (1 - r)$	Use to add a finite geometric series when r is not 1.
Circle form	$(x - h)^2 + (y - k)^2 = r^2$	Read center (h, k) and radius r from the equation.

Model selection

Constant difference suggests arithmetic. Constant ratio suggests geometric or exponential. A maximum or minimum often signals a quadratic model.