

PRACTICE WORKSHEET + FULL ANSWER KEY

Functions, quadratics and transformations

Senior secondary mathematics. Work through the sections in order, or choose the one that matches this week's topic.

QUESTIONS	SECTIONS	ANSWER KEY	WORKING SPACE
104	10	At the back	Ruled

Name _____ Class _____ Date _____

Before you start

Connect each algebraic rule to its graph and domain. Roots, intercepts and turning points answer different questions. State restrictions before simplifying an expression.

WORKED EXAMPLE

Find the roots of $x^2 - 5x + 6 = 0$.

- Find two numbers whose product is 6 and sum is -5: -2 and -3.
- Factor: $(x - 2)(x - 3) = 0$.
- Set each factor to zero: $x = 2$ or $x = 3$.

Use this method as your model. Set out your own working the same way.

Common mistakes to avoid

- Treating $f(x)$ as multiplication
- Cancelling terms without recording excluded values

There are 104 questions across 10 sections. Write in the ruled space, then mark your work against the answer key at the back. Topics covered: Function notation, Quadratics, Exponential models.

SECTION A Function notation, domain and range

$f(x)$ is the output of f for the input x . Record any values that must be excluded.

1

Evaluate $f(3)$ for $f(x) = 2x^2 - 1$.

2

Evaluate $f(-2)$ for $f(x) = x^2 + 3x$.

3

For $f(x) = 3x - 5$, solve $f(x) = 10$.

4

State the domain of $f(x) = 1/(x - 4)$.

5 State the domain of $g(x) = \sqrt{x-3}$.

6 For $f(x) = x^2$ and $g(x) = x + 1$, find $f(g(2))$.

7 For $f(x) = 2x + 1$, find the inverse function.

8 Simplify $(x^2 - 9)/(x - 3)$ and state the restriction.

9 State the range of $f(x) = x^2 + 4$.

10 Is $y^2 = x$ a function of x ? Explain.

11 For $f(x) = 5 - x^2$, find $f(0)$ and $f(3)$.

SECTION B Quadratics

Factorise, complete the square or use the formula. Check the discriminant when you need the number of roots.

12 Find the roots of $x^2 - 5x + 6 = 0$.

13 Factorise $x^2 + x - 12$.

14 Factorise $2x^2 + 7x + 3$.

15 Solve $x^2 - 9 = 0$.

16 Solve $x^2 + 6x + 5 = 0$.

17 Solve $2x^2 - 5x - 3 = 0$.

18 State the vertex of $y = (x - 2)^2 + 5$.

19 Complete the square for $y = x^2 + 6x + 1$.

20 Find the discriminant of $x^2 - 4x + 7 = 0$ and say what it tells you.

21 Solve $x^2 - 4x + 1 = 0$ using the quadratic formula.

22 Find the y-intercept and the x-intercepts of $y = x^2 - 3x - 10$.

23 The graph of $y = x^2 + kx + 9$ touches the x-axis once. Find the positive value of k .

SECTION C Transformations and graphs

Describe each transformation in the order it is applied to the graph of $y = f(x)$.

- 24 Describe the transformation from $y = x^2$ to $y = x^2 + 4$.

- 25 Describe the transformation from $y = x^2$ to $y = (x - 3)^2$.

- 26 Describe the transformation from $y = f(x)$ to $y = -f(x)$.

- 27 Describe the transformation from $y = f(x)$ to $y = 2f(x)$.

- 28 Write the equation of $y = x^2$ after a shift of 2 left and 5 down.

- 29 State the axis of symmetry of $y = x^2 - 8x + 3$.

- 30 State the turning point of $y = x^2 - 8x + 3$ and say whether it is a maximum or a minimum.

- 31 Sketch $y = -(x - 1)^2 + 4$. State the vertex and the direction it opens.

- 32 State the equations of the asymptotes of $y = 1/x$.

- 33 Describe the transformation from $y = f(x)$ to $y = f(x - 2) + 3$.

SECTION D Exponentials, logarithms and indices

A growth factor multiplies; a rate adds. State what every constant means in the model.

- 34 Solve $2^{x+1} = 16$.

- 35 Solve $3^x = 81$.

- 36 Evaluate $\log_2(32)$.

- 37 Write $5^3 = 125$ in logarithm form.

- 38 Simplify $\log(8) + \log(5)$ for logarithms to base 10.

- 39 A population is modelled by $P = 500(1.04)^t$. Explain the 500 and the 1.04.

- 40 For $P = 500(1.04)^t$, find P when $t = 10$, to the nearest whole number.

- 41 A car worth \$24,000 loses 15% of its value each year. Write a model for its value after n years.

- 42 Find the value of that car after 3 years, to the nearest dollar.

- 43 Simplify $(x^3)(x^5)$.

- 44 Simplify $(2x^2)^3$.

SECTION E Applying functions

Interpret the model in context: give units and say what the number means.

- 45 A ball follows $h = -5t^2 + 20t$. Find when it returns to the ground.

- 46 For that model, find the maximum height and when it occurs.

- 47 Revenue is $R = x(60 - x)$. Find the value of x that gives the maximum revenue.

- 48 A rectangle has perimeter 40 m. Write its area as a function of the width w .

- 49 For that rectangle, find the width that gives the largest area.

- 50 Explain why $f(x) = x^2$ has no inverse function unless the domain is restricted.

- 51 A student writes $(x + 3)^2 = x^2 + 9$. Find and correct the error.

- 52 State how many real roots $y = 2x^2 + 3x + 5$ has. Justify your answer.

- 53 Compare $100 + 50t$ with $100(1.5)^t$ when $t = 10$.

- 54 Explain why a growth factor of 0.85 represents a decrease.

SECTION F Polynomials and rational functions*Factorise fully before you solve, and state every excluded value.*

- 55 State the degree of $y = x^3 - 2x$ and the greatest possible number of real roots.

- 56 Factorise $x^3 - 4x$.

- 57 Find the roots of $x^3 - 4x = 0$.

- 58 Describe the end behaviour of $y = -x^4$.

- 59 Find the vertical asymptote of $y = (x + 1)/(x - 5)$.

- 60 Find the horizontal asymptote of $y = (2x + 1)/(x - 3)$.

- 61 Simplify $(x^2 - x - 6)/(x - 3)$ and state the restriction.

- 62 Use the remainder theorem to find the remainder when $x^3 - 2x + 1$ is divided by $x - 2$.

- 63 Show that $x = 1$ is a root of $x^3 - 3x + 2$.

- 64 Solve $x^3 - 3x + 2 = 0$ completely.

SECTION G Trigonometric functions*Sketch the triangle or the unit circle before you write a value.*

- 65 State the exact value of $\sin(30 \text{ degrees})$.

- 66 State the exact value of $\cos(60 \text{ degrees})$.

- 67 State the exact value of $\tan(45 \text{ degrees})$.

- 68 State the period of $y = \sin(x)$ in degrees.

- 69 State the amplitude of $y = 3 \sin(x)$.

70

State the period of $y = \sin(2x)$ in degrees.

71

In a right triangle the opposite side is 3 and the hypotenuse is 5. Find $\sin A$.

72

A right triangle has legs 6 cm and 8 cm. Find the hypotenuse.

73

Solve $\sin(x) = 0.5$ for x between 0 and 180 degrees.

74

State the identity linking $\sin^2(x)$ and $\cos^2(x)$.

SECTION H Sequences and series

Identify whether the sequence is arithmetic or geometric before choosing a formula.

75

Find the 10th term of the arithmetic sequence 3, 7, 11, ...

76

Find the sum of the first 10 terms of 3, 7, 11, ...

77

Find the 6th term of the geometric sequence 2, 6, 18, ...

78

Find the sum of the first 5 terms of 2, 6, 18, ...

79

State the common difference of 20, 16, 12, ...

80

State the common ratio of 81, 27, 9, ...

81

Find the sum to infinity of $8 + 4 + 2 + \dots$

82

State when a geometric series has a sum to infinity.

83

Write the n th term of the arithmetic sequence 5, 8, 11, ...

84

The 3rd term of an arithmetic sequence is 11 and the 7th is 27. Find the first term.

SECTION I Composite and inverse functions

Work from the inside out. The inverse undoes each operation in reverse order.

85 For $f(x) = x + 3$ and $g(x) = 2x$, find $f(g(x))$.

86 For the same functions, find $g(f(x))$.

87 Find the inverse of $f(x) = (x + 4)/3$.

88 Find the inverse of $f(x) = x^3$.

89 State the relationship between the graphs of f and its inverse.

90 A converter is $C(d) = 1.35d$. Find $C(200)$ and say what it means.

91 For that converter, find the inverse and say what it does.

92 A cost is $C(n) = 40 + 6n$. Find n when $C(n) = 148$.

93 State the domain of $f(x) = \ln(x - 2)$.

94 Explain why $f(g(x))$ and $g(f(x))$ are usually different.

SECTION J Mixed review and challenge

Match each model to its graph, and check that the answer makes sense in context.

95 Solve $3^{2x} = 27$.

96 Solve $\log_3(x) = 4$.

97 Solve $x^2 - 2x - 8 \leq 0$.

98 Sketch $y = 2^x$ and state its y -intercept and asymptote.

99 A ball follows $h = 30t - 5t^2$. Find the times when $h = 25$.

100 A rectangle has area 48 m^2 and length x . Write the perimeter as a function of x .

101 Explain why $y = 3^x$ has no x -intercept.

102

A quantity halves every 5 years from a start of 200.
Write a model for the amount after t years.

103

For that model, find the amount after 15 years.

104

A student writes $\log(a + b) = \log(a) + \log(b)$. Correct the error.

Answer key

Work each section before you check it. These are model answers: an equivalent method, or a different well-supported interpretation, can be just as correct.

SECTION A - FUNCTION NOTATION, DOMAIN AND RANGE

1. 17.
2. -2.
3. $x = 5$.
4. All real x except $x = 4$.
5. $x \geq 3$.
6. 9.
7. $f^{-1}(x) = (x - 1)/2$.
8. $x + 3$, with x not equal to 3.
9. $f(x) \geq 4$.
10. No. One value of x can give two values of y .
11. $f(0) = 5$ and $f(3) = -4$.

SECTION B - QUADRATICS

12. $x = 2$ or $x = 3$.
13. $(x + 4)(x - 3)$.
14. $(2x + 1)(x + 3)$.
15. $x = 3$ or $x = -3$.
16. $x = -1$ or $x = -5$.
17. $x = 3$ or $x = -1/2$.
18. (2, 5).
19. $y = (x + 3)^2 - 8$.
20. -12, so the equation has no real roots.
21. $x = 2 \pm \sqrt{3}$.
22. y -intercept (0, -10); x -intercepts (5, 0) and (-2, 0).
23. $k = 6$, because the discriminant $k^2 - 36$ must be zero.

SECTION C - TRANSFORMATIONS AND GRAPHS

24. A translation 4 units up.
25. A translation 3 units to the right.
26. A reflection in the x -axis.
27. A vertical stretch with scale factor 2.
28. $y = (x + 2)^2 - 5$.
29. $x = 4$.
30. (4, -13); a minimum, because the coefficient of x^2 is positive.
31. Vertex (1, 4); the parabola opens downwards.
32. $x = 0$ and $y = 0$.
33. A translation 2 units right and 3 units up.

SECTION D - EXPONENTIALS, LOGARITHMS AND INDICES

34. $x = 3$.
35. $x = 4$.

36. 5.
37. $\log_5(125) = 3$.
38. $\log(40)$, which is about 1.602.
39. 500 is the initial population; 1.04 is the growth factor, a 4% increase each time period.
40. About 740.
41. $V = 24000(0.85)^n$.
42. \$14,739.
43. x^8 .
44. $8x^6$.

SECTION E - APPLYING FUNCTIONS

45. At $t = 4$ seconds, since $h = 0$ when $t = 0$ or $t = 4$.
46. 20 m, at $t = 2$ seconds.
47. $x = 30$, giving a revenue of 900.
48. $A = w(20 - w)$.
49. $w = 10$ m, a square of area 100 m².
50. Two different inputs give the same output, so the reverse mapping is not a function. Restricting to $x \geq 0$ fixes this.
51. The middle term is missing: $(x + 3)^2 = x^2 + 6x + 9$.
52. None. The discriminant is $9 - 40 = -31$, which is negative.
53. The linear model gives 600; the exponential model gives about 5,767, so exponential growth is far faster.
54. Each value is 85% of the previous one, so the quantity falls by 15% per period.

SECTION F - POLYNOMIALS AND RATIONAL FUNCTIONS

55. Degree 3, with up to three real roots.
56. $x(x - 2)(x + 2)$.
57. $x = 0$, $x = 2$ and $x = -2$.
58. y falls without limit as x goes to either positive or negative infinity.
59. $x = 5$.
60. $y = 2$.
61. $x + 2$, with x not equal to 3.
62. 5, because $f(2) = 5$.
63. $f(1) = 1 - 3 + 2 = 0$, so $x = 1$ is a root.
64. $x = 1$ (a repeated root) and $x = -2$.

SECTION G - TRIGONOMETRIC FUNCTIONS

65. $1/2$.
66. $1/2$.
67. 1.

- 68. 360 degrees.
- 69. 3.
- 70. 180 degrees.
- 71. $\frac{3}{5}$.
- 72. 10 cm.
- 73. $x = 30$ degrees or $x = 150$ degrees.
- 74. $\sin^2(x) + \cos^2(x) = 1$.

SECTION H - SEQUENCES AND SERIES

- 75. 39.
- 76. 210.
- 77. 486.
- 78. 242.
- 79. -4.
- 80. $\frac{1}{3}$.
- 81. 16.
- 82. When the common ratio r satisfies $-1 < r < 1$.
- 83. $3n + 2$.
- 84. 3, because the common difference is 4.

SECTION I - COMPOSITE AND INVERSE FUNCTIONS

- 85. $2x + 3$.
- 86. $2x + 6$.
- 87. $f^{-1}(x) = 3x - 4$.
- 88. $f^{-1}(x) = x^{1/3}$.
- 89. They are reflections of each other in the line $y = x$.
- 90. 270; 200 units convert to 270 units.
- 91. $C^{-1}(x) = x/1.35$; it converts back to the original currency.
- 92. $n = 18$.
- 93. $x > 2$.
- 94. Composition is not commutative; changing the order changes which operation is applied first.

SECTION J - MIXED REVIEW AND CHALLENGE

- 95. $x = \frac{3}{2}$.
- 96. $x = 81$.
- 97. $-2 \leq x \leq 4$.
- 98. y-intercept (0, 1) and horizontal asymptote $y = 0$.
- 99. $t = 1$ second and $t = 5$ seconds.
- 100. $P = 2x + 96/x$.
- 101. 3^x is always positive, so the graph never reaches $y = 0$.
- 102. $A = 200(0.5)^{t/5}$.
- 103. 25.
- 104. $\log(a) + \log(b) = \log(ab)$. A logarithm of a sum cannot be expanded that way.

Review and next steps

Mark your work against the answer key, record your score for each section, then answer the three questions below. Bring this page to your next lesson so your tutor can start from what actually blocked you.

Which question did you find hardest, and what exactly stopped you?

Rewrite the corrected working for that question here.

Which section do you want to practise again before your next lesson?

Score record

SECTION	QUESTIONS	NUMBER CORRECT	PRACTISE AGAIN?
A. Function notation, domain and range	11		
B. Quadratics	12		
C. Transformations and graphs	10		
D. Exponentials, logarithms and indices	11		
E. Applying functions	10		
F. Polynomials and rational functions	10		
G. Trigonometric functions	10		
H. Sequences and series	10		
I. Composite and inverse functions	10		
J. Mixed review and challenge	10		
Total	104		

Keep going with one-to-one support.

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