

AP Calculus AB Practice Worksheet

Fifty questions on limits, derivative rules, applications of the derivative, integrals, and motion. Questions 45 to 50 ask for written working.

QUESTIONS	50	MULTIPLE CHOICE	7	WRITTEN	43
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Instructions

Show every step. A calculator is fine for arithmetic, not for the calculus itself.

1 Find $\lim_{x \rightarrow 3} \frac{(x^2 - 9)}{(x - 3)}$.

2 Find $\lim_{x \rightarrow 0} \frac{(\sin x)}{x}$.

3 Find $\lim_{x \rightarrow \infty} \frac{(3x + 1)}{(x - 2)}$.

4 A function f is continuous at $x=2$ if:

- A) $f(2)$ exists only
- B) $\lim_{x \rightarrow 2} f(x)$ exists only
- C) $\lim_{x \rightarrow 2} f(x)$ equals $f(2)$
- D) f is differentiable at 2

5 Differentiate: $f(x) = 5x^3 - 2x + 7$.

6 Differentiate: $f(x) = x^4 + 3x^2 - 1$.

7 Differentiate: $f(x) = x^2 e^x$. (product rule)

8 Differentiate: $f(x) = \frac{\sin(x)}{x}$. (quotient rule)

9 Differentiate: $f(x) = (3x + 1)^5$. (chain rule)

10 Differentiate: $f(x) = \sin(x^2)$.

11 Differentiate: $f(x) = \ln(x^2 + 1)$.

12 Differentiate: $f(x) = e^{3x}$.

- 13** Find $f'(2)$ if $f(x) = x^3$.

- 14** What does $f'(x) = 0$ at a point usually indicate?
- A) An asymptote
 - B) A possible extremum
 - C) A discontinuity
 - D) f is undefined

- 15** Find the critical points of $f(x) = x^2 - 4x + 3$.

- 16** Find the critical points of $f(x) = x^3 - 12x$.

- 17** If $f''(x) > 0$ at a critical point, the point is a:
- A) Local max
 - B) Local min
 - C) Inflection point
 - D) Not classifiable

- 18** If $f''(x) < 0$ at a critical point, the point is a:
- A) Local max
 - B) Local min
 - C) Inflection point
 - D) Not classifiable

- 19** A square's side grows at 2 cm/s. How fast is the area growing when the side is 5 cm? ($dA/dt = 2s \, ds/dt$)

- 20** Find the equation of the tangent line to $f(x) = x^2$ at $x = 3$.

- 21** Evaluate: $\int x^4 \, dx$.

- 22** Evaluate: $\int (3x^2 - 2x) \, dx$.

- 23** Evaluate: $\int_0^3 2x \, dx$.

- 24** Evaluate: $\int_1^4 3 \, dx$.

- 25** Evaluate: $\int_0^2 (x^2 + 1) \, dx$.

- 26** Use u-substitution to evaluate: $\int 2x(x^2+1)^3 \, dx$.

- 27** Find the area between $y = x$ and $y = x^2$ on $[0, 1]$.

28 The Fundamental Theorem of Calculus connects:

- A) Limits and continuity
- B) Derivatives and integrals
- C) Series and sequences
- D) Related rates and optimization

29 If $\frac{d}{dx} \int_0^x f(t) dt = x^2 + 1$, what is $f(x)$?

30 A particle's position is $s(t) = t^3 - 6t^2$. Find its velocity function.

31 Using the same $s(t)$, find the acceleration function.

32 A particle's velocity is $v(t) = 2t - 4$. When is the particle at rest?

33 Find the average value of $f(x) = x^2$ on $[0, 3]$.

34 Which is the correct form of the chain rule for $f(g(x))$?

- A) $f'(x) g'(x)$
- B) $f'(g(x))$
- C) $f'(g(x)) g'(x)$
- D) $f(g'(x))$

35 Solve the separable equation $dy/dx = 2y$, given $y(0) = 5$.

36 Find the derivative of $f(x) = \sqrt[3]{x}$. (write as a power first)

37 Find the second derivative of $f(x) = x^4$.

38 A function's graph is concave up where:

- A) $f' > 0$
- B) $f' < 0$
- C) $f'' > 0$
- D) $f'' < 0$

39 Find any inflection points of $f(x) = x^3$.

40 A rectangle's perimeter is fixed at 20. Write its area as a function of one side, x . ($A(x)$, other side = $10-x$)

41 Using the function above, find the width that maximizes area.

- 42** Evaluate: $\lim_{x \rightarrow 0} (e^x - 1)/x$. (use L'Hopital's rule)

- 43** Evaluate: $\lim_{x \rightarrow \infty} (\ln x)/x$.

- 44** Differentiate implicitly: find dy/dx if $x^2 + y^2 = 25$.

- 45** A balloon's radius grows at 3 cm/s. Find dV/dt when $r = 4$ ($V = 4/3 \pi r^3$, $dV/dt = 4 \pi r^2 dr/dt$), and explain each factor in the formula.

- 46** Explain why a function can have a critical point that is NEITHER a maximum nor a minimum, and give an example.

- 47** Explain the difference between average velocity and instantaneous velocity, and how each relates to a position function.

- 48** A tank drains so that its volume is $V(t) = 500 - 20t$. Explain what the derivative $V'(t)$ represents in this context.

- 49** Explain why $\int_a^b f(x)dx$ can be negative even though the integral represents an 'area'.

- 50** A cost function is $C(x) = 0.01x^2 + 5x + 200$. Find the marginal cost at $x=100$ and explain what 'marginal cost' means as a derivative.

Answer key

Direct answers only, in question order. Work the set first, then check.

1. 6	2. 1	3. 3	4. C) $\lim_{x \rightarrow 2} f(x)$ as $x \rightarrow 2$ equals $f(2)$	5. $15x^2 - 2$
6. $4x^3 + 6x$	7. $2x \times e^x + x^2 \times e^x$	8. $(x \times \cos(x) - \sin(x))/x^2$	9. $15(3x+1)^4$	10. $2x \times \cos(x^2)$
11. $2x/(x^2+1)$	12. $3e^{3x}$	13. 12	14. B) A possible extremum	15. $x = 2$
16. $x = 2$ and $x = -2$	17. B) Local min	18. A) Local max	19. 20	20. $y = 6x - 9$
21. $x^5/5 + C$	22. $x^3 - x^2 + C$	23. 9	24. 9	25. $14/3$
26. $(x^2+1)^4/4 + C$	27. $1/6$	28. B) Derivatives and integrals	29. $x^2 + 1$	30. $3t^2 - 12t$
31. $6t - 12$	32. $t = 2$	33. 3	34. C) $f'(g(x)) g'(x)$	35. $y = 5e^{2x}$
36. $1/(2 \times \sqrt{x})$	37. $12x^2$	38. C) $f'' > 0$	39. $x = 0$	40. $A(x) = x(10 - x)$
41. 5	42. 1	43. 0	44. $dy/dx = -x/y$	45. $dV/dt = 192 \times \pi \text{ cm}^3/\text{s}$; $4 \times \pi \times r^2$ is the surface area factor from differentiating volume, and dr/dt is the given rate
46. If f' does not change sign at the critical point, it is an inflection point rather than an extremum; $f(x) = x^3$ at $x=0$ is an example since $f'(x) = 3x^2 \geq 0$ on both sides	47. Average velocity is the slope of the secant line, $(s(b)-s(a))/(b-a)$; instantaneous velocity is the derivative $s'(t)$, the slope of the tangent line at a single moment	48. $V'(t) = -20$ represents the rate the tank is draining, 20 units of volume per unit time, with the negative sign showing volume is decreasing	49. When $f(x)$ is below the x-axis on the interval, the signed area is negative, since the definite integral measures net signed area, not unsigned geometric area	50. $C'(x) = 0.02x + 5$, so $C'(100) = 7$; marginal cost is the instantaneous rate of change of cost with respect to one more unit produced

Error log

Bring this page to your next session so the tutor can start from what actually blocked you.

Questions I answered incorrectly:

Was it the concept, the setup, or the arithmetic?

The single topic I want to start the next session with:
