

AP Calculus BC Practice Worksheet

Fifty questions covering the full BC scope: advanced integration, parametric and polar functions, sequences and series, and Taylor polynomials. Questions 45 to 50 ask for written working.

QUESTIONS	50	MULTIPLE CHOICE	12	WRITTEN	38
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Instructions

Identify the series or curve type before choosing a technique. Show every step.

1 Evaluate: $\int x \cos(x) dx$ using integration by parts.

2 Evaluate: $\int \ln(x) dx$ using integration by parts.

3 Evaluate: $\int_0^1 x e^x dx$.

4 Integration by parts is the reverse of which rule?

- A) Chain rule
- B) Product rule
- C) Quotient rule
- D) Power rule

5 Split into partial fractions: $1/((x-1)(x+1))$.

6 Evaluate: $\int_1^{\infty} 1/x^2 dx$.

7 Does $\int_1^{\infty} 1/x dx$ converge or diverge?

- A) Converges
- B) Diverges
- C) Equals 0
- D) Cannot determine

8 Find dy/dx for $x(t) = 3t$, $y(t) = t^2$.

9 Find dy/dx for $x(t) = t^3$, $y(t) = t^2 + 1$.

10 Find the speed at $t=1$ for $x(t) = t^2$, $y(t) = t^3$ (speed = $\sqrt{(x')^2 + (y')^2}$).

11 A polar curve's area formula uses which exponent on r ?

- A) 1
- B) 2
- C) $1/2$
- D) 3

- 12 Find the area enclosed by $r = 2$ for $0 \leq \theta \leq 2\pi$.

- 13 Which test is the fastest first check for series divergence?

- A) Ratio test
- B) nth term test
- C) Integral test
- D) Comparison test

- 14 Does $\sum (n/(n+1))$ from $n=1$ to infinity converge or diverge? (use the nth term test)

- 15 Find the sum of the geometric series $\sum (1/3)^n$ from $n=1$ to infinity.

- 16 Find the sum of the geometric series $\sum 4(1/2)^n$ from $n=0$ to infinity.

- 17 A p-series $\sum 1/n^p$ converges when:

- A) $p < 1$
- B) $p = 1$
- C) $p > 1$
- D) Never converges

- 18 Does $\sum 1/n$ (the harmonic series) converge?

- A) Yes
- B) No
- C) Only for even n
- D) Cannot determine

- 19 Use the ratio test setup: for $\sum x^n/n!$, find $\lim |a_{n+1}/a_n|$ in terms of x . (this determines convergence for all x)

- 20 An alternating series $\sum (-1)^n / n$ converges by which test?

- A) Ratio test
- B) Alternating series test
- C) p-series test
- D) Comparison test

- 21 Write the first three nonzero terms of the Maclaurin series for e^x .

- 22 Write the first three nonzero terms of the Maclaurin series for $\sin(x)$.

- 23 Find the 2nd-degree Taylor polynomial for $f(x) = \ln(x)$ centered at $a=1$. ($f(1)=0$, $f'(1)=1$, $f''(1)=-1$)

- 24 A Maclaurin series is a Taylor series centered at:

- A) $x = 1$
- B) $x = -1$
- C) $x = 0$
- D) $x = a$ (any point)

- 25 Given $f(x)=\cos(x)$, find the 4th-degree Maclaurin polynomial.

26 Euler's method approximates solutions to:

- A) Definite integrals
- B) Differential equations
- C) Series sums
- D) Limits

27 Using Euler's method with $h=0.5$, $y(0)=1$, and $dy/dx = y$, find the approximation for $y(0.5)$.

28 Logistic growth $dP/dt = kP(1 - P/L)$ slows down as P approaches:

- A) 0
- B) k
- C) L
- D) Infinity

29 A logistic model has carrying capacity $L=1000$. As $t \rightarrow \infty$, what does $P(t)$ approach?

30 Find arc length setup (do not evaluate): the integral for the length of $y=x^2$ from $x=0$ to $x=1$.

31 Evaluate: $\int (2x+1)/(x^2+x) dx$. (hint: the numerator is the derivative of the denominator)

32 Find the interval of convergence type for $\sum x^n/n$ (name the general shape: interval centered where?)

33 Which of these is a BC-only topic not covered in AP Calculus AB?

- A) Chain rule
- B) Related rates
- C) Taylor polynomials
- D) Definite integrals

34 Find velocity vector at $t=2$ for $x(t)=t^2$, $y(t)=3t$.

35 A particle moves along $x(t)=\cos(t)$, $y(t)=\sin(t)$. Describe the path.

36 Evaluate: $\int x^2 e^x dx$. (integration by parts, applied twice)

37 Determine whether $\sum (-1)^n/n^2$ converges absolutely, conditionally, or diverges.

38 Determine whether $\sum (-1)^n/n$ converges absolutely, conditionally, or diverges.

39 The Lagrange error bound estimates:

- A) The exact value of a series
- B) The error in a Taylor approximation
- C) The radius of convergence
- D) The derivative of a function

- 40** A slope field shows $dy/dx = x$. Describe the general shape of the solution curves.

- 41** Explain why the ratio test is especially useful for series involving factorials, using $\sum 1/n!$ as context.

- 42** Explain the geometric meaning of $dy/dx = (dy/dt)/(dx/dt)$ for a parametric curve.

- 43** A tank's volume follows logistic growth toward a capacity of 500 gallons. Explain why the growth rate is fastest at $P=250$, not at $P=0$.

- 44** Explain why $\sum 1/n$ diverges even though its terms approach 0, contrasting it with $\sum 1/n^2$ which converges.

- 45** A curve is given parametrically by $x(t)=t-\sin(t)$, $y(t)=1-\cos(t)$ (a cycloid). Explain, without computing, why dy/dx is undefined when $t=0$.

- 46** Explain why a 2nd-degree Taylor polynomial usually approximates a function better near the center than a 1st-degree (tangent line) approximation does.

- 47** Find the sum of the geometric series $\sum 3(1/4)^n$ from $n=1$ to infinity.

- 48** Use the ratio test to determine whether $\sum n/2^n$ converges or diverges.

- 49** Find the acceleration vector at $t=1$ for $x(t)=t^3$, $y(t)=t^2$. ($a = \langle x''(t), y''(t) \rangle$)

- 50** For the polar curve $r = 1 + \cos(\theta)$, find r at $\theta = 0$.

Answer key

Direct answers only, in question order. Work the set first, then check.

1. $x \sin(x) + \cos(x) + C$	2. $x \ln(x) - x + C$	3. 1	4. B) Product rule	5. $\frac{1}{2(x-1)} - \frac{1}{2(x+1)}$
6. 1	7. B) Diverges	8. $2t/3$	9. $2/(3t)$	10. $\sqrt[3]{4+9} = \sqrt[3]{13}$
11. B) 2	12. 4π	13. B) nth term test	14. Diverges, since the terms approach 1, not 0	15. 1/2
16. 8	17. C) $p > 1$	18. B) No	19. 0 for all x, so the series converges for all x	20. B) Alternating series test
21. $1 + x + x^2/2$	22. $x - x^3/6 + x^5/120$	23. $(x-1) - (x-1)^2/2$	24. C) $x = 0$	25. $1 - x^2/2 + x^4/24$
26. B) Differential equations	27. 1.5	28. C) L	29. 1000	30. $\int_0^1 \frac{1}{\sqrt{1+4x^2}} dx$
31. $\ln x^2+x + C$	32. centered at $x=0$	33. C) Taylor polynomials	34. $\langle 4, 3 \rangle$	35. a unit circle traced counterclockwise
36. $x^2 e^x - 2x e^x + 2x e^x + C$	37. Converges absolutely, since $\sum 1/n^2$ is a convergent p-series with $p=2$	38. Converges conditionally, since the alternating series test applies but $\sum 1/n$ diverges	39. B) The error in a Taylor approximation	40. parabolas of the form $y = x^2/2 + C$
41. The ratio $a_{n+1}/a_n = 1/(n+1)$ simplifies cleanly for factorials, and its limit of 0 immediately confirms convergence	42. It is the slope of the tangent line to the curve, found by dividing the rate of change of y by the rate of change of x, both with respect to the parameter t	43. The rate $kP(1-P/L)$ is a downward parabola in P, maximized at $P=L/2=250$, since growth is limited both by scarcity near $P=0$ and by the shrinking $(1-P/L)$ factor near $P=L$	44. Passing the nth term test is necessary but not sufficient for convergence; the harmonic series' terms shrink too slowly for the partial sums to stay bounded, while $1/n^2$ shrinks fast enough	45. Both dx/dt and dy/dt equal 0 at $t=0$, so the ratio $dy/dx = 0/0$ is indeterminate there

46. The 2nd-degree polynomial matches the function's value, slope, AND concavity at a , capturing curvature that a straight tangent line cannot

47. 1

48. Converges, since the limit of the ratio is $1/2$, which is less than 1

49. $\langle 6, 2 \rangle$

50. 2

Error log

Bring this page to your next session so the tutor can start from what actually blocked you.

Questions I answered incorrectly:

Was it the concept, the setup, or the arithmetic?

The single topic I want to start the next session with:
