

# AP Precalculus Formula and Strategy Guide

The three AP Precalculus units College Board assesses: polynomial and rational functions, exponential and logarithmic functions, and trigonometric and polar functions — with the modeling and rate-of-change reasoning the exam rewards.

|          |   |                 |   |        |                |
|----------|---|-----------------|---|--------|----------------|
| SECTIONS | 4 | WORKED EXAMPLES | 3 | FORMAT | Print friendly |
|----------|---|-----------------|---|--------|----------------|

## How to use this guide

A formula you cannot place is not knowledge yet. Work the recall drill below before the questions.

- AP Precalculus scores explanations, not just answers — practice stating WHY a function behaves a certain way.
- Rate of change over an interval is a recurring AP Precalculus theme; know average rate of change cold.
- Sketch first. A rough graph catches sign errors before they become wrong answers.

## 1. Polynomial and rational functions

| NAME                           | FORMULA                                                             | WHEN TO USE                                             |
|--------------------------------|---------------------------------------------------------------------|---------------------------------------------------------|
| Average rate of change         | $(f(b) - f(a)) / (b - a)$                                           | Comparing how fast a function changes over an interval. |
| End behavior (polynomial)      | determined by the leading term                                      | Predicting far-left/far-right behavior.                 |
| Local extrema                  | where a function changes from increasing to decreasing (or reverse) | Locating a graph's peaks and valleys.                   |
| Vertical asymptote             | denominator = 0, numerator does not cancel                          | A rational function's undefined behavior.               |
| Removable discontinuity (hole) | a factor cancels top and bottom                                     | A rational function looks undefined but has a limit.    |
| End behavior (rational)        | compare degrees of numerator and denominator                        | A rational function's long-run behavior.                |

### WORKED EXAMPLE

For  $f(x) = x^2$  on  $[1, 4]$ , find the average rate of change.

1.  $f(4) = 16, f(1) = 1$
2.  $(16 - 1) / (4 - 1) = 15/3$
3.  $= 5$

Check: The secant line from  $(1,1)$  to  $(4,16)$  has slope 5.

## 2. Exponential and logarithmic functions

| NAME                     | FORMULA                                               | WHEN TO USE                                          |
|--------------------------|-------------------------------------------------------|------------------------------------------------------|
| Exponential growth       | $y = a \times b^x, b > 1$                             | A constant percentage increase over equal intervals. |
| Exponential decay        | $y = a \times b^x, 0 < b < 1$                         | A constant percentage decrease over equal intervals. |
| Key exponential property | equal input changes $\rightarrow$ equal output RATIOS | Recognizing exponential behavior from a table.       |
| Logarithm as inverse     | $\log_b(y) = x \Leftrightarrow b^x = y$               | Undoing an exponential relationship.                 |
| Semi-log linearity       | $\log(y)$ is linear in $x$ for exponential $y$        | Confirming exponential data using logs.              |

### WORKED EXAMPLE

A table shows y-values 3, 6, 12, 24 for equally spaced x-values. What kind of function is this?

1. Check ratios:  $6/3 = 2$ ,  $12/6 = 2$ ,  $24/12 = 2$
2. Constant ratio for equal input steps means exponential

Check: Exponential functions have constant output RATIOS, not constant differences.

## 3. Trigonometric and polar functions

| NAME                 | FORMULA                                            | WHEN TO USE                                   |
|----------------------|----------------------------------------------------|-----------------------------------------------|
| Sinusoidal function  | $y = a \sin(b(x-h)) + k$                           | Modeling periodic behavior.                   |
| Amplitude            | $ a $                                              | The height of a wave above/below its midline. |
| Period               | $2\pi/ b $                                         | How often a periodic function repeats.        |
| Polar coordinates    | $(r, \theta)$ : $r$ is distance, $\theta$ is angle | Describing a point by distance and direction. |
| Polar to rectangular | $x = r \cos(\theta), y = r \sin(\theta)$           | Converting between coordinate systems.        |

### WORKED EXAMPLE

Find the period of  $y = 3 \sin(2x) + 1$ .

1. Period formula:  $2\pi/|b|$
2.  $b = 2$
3. Period =  $2\pi/2 = \pi$

Check: Doubling the input rate halves the period.

## 4. Function modeling

| NAME | FORMULA | WHEN TO USE |
|------|---------|-------------|
|------|---------|-------------|

| NAME                            | FORMULA                                                                                    | WHEN TO USE                                           |
|---------------------------------|--------------------------------------------------------------------------------------------|-------------------------------------------------------|
| Choosing a model                | linear (constant difference), exponential (constant ratio), quadratic/polynomial otherwise | Matching a data pattern to a function family.         |
| Concavity                       | increasing rate = concave up; decreasing rate = concave down                               | Describing how a rate of change itself is changing.   |
| Composite functions in modeling | combining two processes into one function                                                  | One quantity depends on another that depends on $x$ . |

## Notes and tutor hand-off

---

Bring this page to your next session so the tutor can start from what actually blocked you.

**Which unit (functions, exponential/log, or trig/polar) needs more review?**

---

---

---

**One rate-of-change idea you want to practice explaining out loud:**

---

---

---

**Worksheet question numbers to review with a tutor:**

---

---

---